INFORMS International - Israel

Forecasting and Tradeoff Analysis Using DEA: With Applications to Software Engineering Management

David N. Reese and Joseph C. Paradi

July 1, 1998

Presentation Overview

- *Introduction and motivation*
- *Summary of relevant literature*
- *Tradeoff analysis using DEA*
- *Generating initial forecasts*
- *Applications to software engineering management*
- *Summary and Future Work*

Introduction

- *Poor forecasting and estimation can be the root cause of many software project schedule and cost overruns*
- *Current forecasting and estimating tools for software project management:*
  - *various proprietary and non-proprietary parametric models available - COCOMO*
  - *Expert judgement: experience; rules of thumb*
- *Our research objective is to apply non-parametric methods for forecasting and tradeoff analysis*

Summary of Literature

- *DEA research has focused mainly on the assessment and control of past performance*
- *A few researches have applied DEA for predictive purposes:*
  - *Bank failure prediction (Barr, Seiford and Siems 1994 & Siems 1992)*
  - *Performance of DMUs which do not yet exist (Golany 1993)*
  - *Credit Union failure prediction (Pille and Paradi 1997)*
  - *Corporate failure prediction (Paradi and Simak 1997)*
- *Little work has been done on applying DEA for forecasting and tradeoff analysis*

Tradeoff Analysis & Marginal Rates

- *In the DEA and economic literature marginal rates are slopes or partial derivatives on the efficient frontier*
- *Marginal rates are a special case of tradeoffs limited to assessing the impacts of infinitesimal changes of one or more variables on one or more other variables*
- *Ratios of DEA multipliers can provide marginal rates*
- *Problem: DEA multipliers may not be unique*

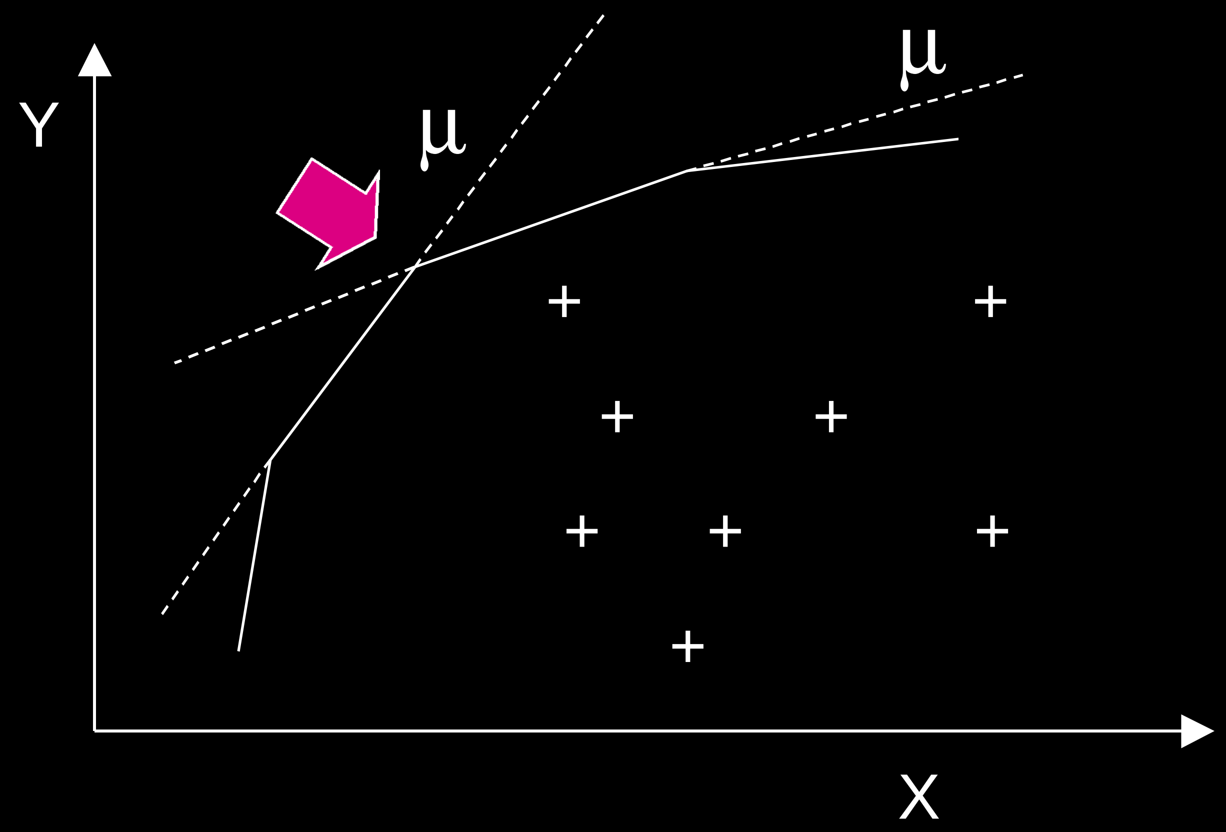
Tradeoffs etc. Cont'd...

- *Rosen et al. 1995 address this problem and present a general framework for the computation of marginal rates in DEA*
- *Goal: adapt this framework for interactively analysing general tradeoffs (beyond marginal rates)*
- *Specifically, the finite difference approach presented in Rosen et al. is extended to allow managers to examine various scenarios and tradeoffs among the project's objectives.*

Tradeoff Analysis Using DEA

- *The calculation of marginal rates at any given point on the frontier is limited to providing the change on one or more throughputs by modifying one or more other throughputs by a **small infinitesimal** change.*
- *But management may want to make a **much larger** change than is possible with this.*
- *Marginal rates vary widely from point to point on the frontier - can't just extrapolate*
- *So new methods are needed to handle this*

Example: 2-D Envelopment



More on Tradeoffs and DEA

- *We first consider **pairwise tradeoffs** - i.e. obtain the* i-*th throughput that results from the increasing or decreasing the* j-*th throughput by* h.
- *A finite differences approach can be adapted to calculate pairwise tradeoffs*
- *It is necessary to add a slack to ensure that the increment* h *in the j-th throughput does not fall outside of the realm of production possibilities.*

Some Notation

- $\boldsymbol{z^T = (y^T, -x^T)}$ *- a vector of throughputs (a.k.a netputs)*
- ***Z*** *- a matrix of throughputs*
- h *- scalar or additive increment*
- I, O - *set of input and output indices*
- ***u***, ***v*** *- unit directional vectors*
- A - *set of throughputs modified by* α *units (specified by direction* ***v****)*
- B - *set of throughputs modified by* β *units (specified by direction* ***u****)*
- ε - *a non-Archimedian infinitesimal*

Pairwise Tradeoffs (Eq "A")

- *Get* i-*th throughput that results from increasing or decreasing the* j-*th by* h: $z_{j0}' = z_{j0} \pm h$

$$
\begin{aligned}
\max_{l,l_0,z_0',s} \quad & -s + e(z_{i0}') \\
s.t. \quad & \mathbf{z}_0 l_0 + \mathbf{Z}\lambda \geq \mathbf{z}_0' \\
& l_0 + \mathbf{1}^T \lambda = 1 \\
& z_{l0}' = z_{l0}, \quad l \neq i, j \\
& z_{j0}' + s = z_{j0} + h \\
& \lambda_0, \lambda \geq \mathbf{0}, \mathbf{z}_0' \geq 0, \\
& s \geq 0 \text{ if } h > 0 \text{ and } s \leq 0 \text{ if } h < 0
\end{aligned}
$$

Scalar Tradeoffs (EQ "B")

$$\max_{l, l_0, z_0', s} \quad -s + e(z_{i0}')$$

$$s.t. \quad \mathbf{z}_0 l_0 + \mathbf{Z}\lambda \geq \mathbf{z}_0'$$

$$l_0 + \mathbf{1}^T \lambda = 1$$

$$z_{l0}' = z_{l0}, \quad l \neq i, j$$

$$z_{j0}' + s = z_{j0}(h)$$

$$\lambda_0, \lambda \geq \mathbf{0}, \mathbf{z}_0' \geq 0,$$

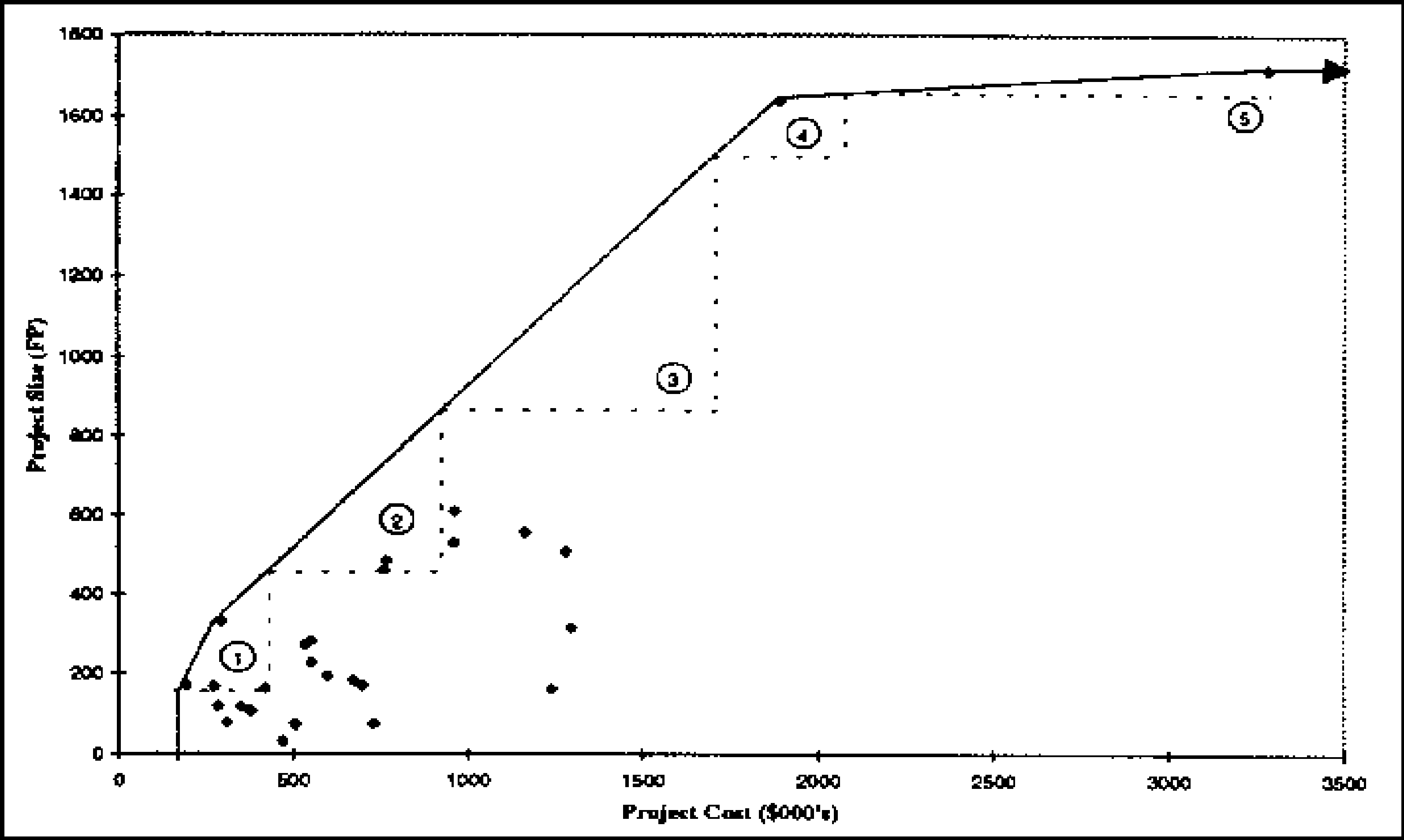
$$\text{if } h > 1, s \leq 0 \text{ if } j \in I \text{ and } s \geq 0 \text{ if } j \in O$$

$$\text{if } h < 1, s \geq 0 \text{ if } j \in I \text{ and } s \leq 0 \text{ if } j \in O$$

Some Generalizations

- *In some situations it is necessary to assess the impacts of changes in one or more throughputs on one or more other throughputs*
- *E.g. the impact of a 10% increase in software size on cost and quality while holding the time to market constant*
- *Scalar and additive generalized models have been developed (see Appendix)*

A Real Example



Generating Initial Forecasts

- The following model determines the minimum cost x of producing outputs y_0 (size, quality, and duration)
- Provides the starting point for tradeoff analysis

$$
\begin{aligned}
\min_{x,\lambda} \quad & x \\
s.t. \quad & \mathbf{Y}\lambda \geq \mathbf{y}_0 \\
& x - \mathbf{X}\lambda \geq \mathbf{0} \\
& x, \lambda \geq 0
\end{aligned}
$$

Apply to Project Management

1) Define and prioritise the project objectives
2) Generate an initial (efficient) project forecast
3) Generate alternatives using tradeoff analysis tools
4) Select the best alternative
5) Produce or revise project plan
6) Monitor project and project environment (go to step 1 if necessary)

Summary and Future Work

- *Several methods for calculating general tradeoffs have been presented*
  - *pairwise additive and scalar methods, with generalisations*
- *Discussed applications to software engineering management*

- *Future work:*
  - *incorporate uncertainty and probability into models*
  - *develop general methods for generating initial forecasts*
  - *add constraints representing allowable ranges of inputs and outputs*
  - *explore relationship of generalised scalar model with RTS measures*

Appendix A

- *Generalized Additive Tradeoffs*
- *Generalized Scalar Tradeoffs*

Generalised Additive Tradeoffs

$$\max_{l, l_0, z_0', s} \quad -\sum_j s_j + e(a), \quad \forall j \in B$$

$$s.t. \quad \mathbf{z}_0 l_0 + \mathbf{Z}\lambda \geq \mathbf{z}_0'$$

$$l_0 + \mathbf{1}^T \lambda = 1$$

$$z_{j0}' + s_j = z_{j0} + b, \quad \forall j \in B$$

$$z_{i0}' - a = z_{i0}, \quad \forall i \in A$$

$$z_{k0}' = z_{k0}, \quad \forall k \in K$$

$$\lambda_0, \lambda \geq \mathbf{0}, \mathbf{z}_0' \geq 0,$$

$$\mathbf{s} \geq \mathbf{0} \text{ if } b > 0 \text{ and } \mathbf{s} \leq \mathbf{0} \text{ if } b < 0$$

k is not contained in either A or B

Generalised Scalar Tradeoffs

$$\max_{l, l_0, z_0', s} \; -\sum_j s_j + e(a), \quad \forall j \in B$$

$$
\begin{aligned}
s.t. \quad & \mathbf{z}_0 l_0 + \mathbf{Z}\lambda \geq \mathbf{z}_0' \\
& l_0 + \mathbf{1}^T \lambda = 1 \\
& z_{j0}' + s_j = z_{j0}(b), \quad \forall j \in (B \cap O) \\
& z_{j0}' + s_j = z_{j0}(2 - b), \quad \forall j \in (B \cap I) \\
& z_{i0}' - z_{i0}(a) = 0, \quad \forall i \in (A \cap O) \\
& z_{i0}' - z_{i0}(2 - a) = 0, \quad \forall i \in (A \cap I) \\
& z_{k0}' = z_{k0}, \quad \forall k \in K \\
& \lambda_0, \lambda \geq \mathbf{0}, \mathbf{z}_0' \geq 0, \\
& \text{if } b > 1, \mathbf{s} \geq \mathbf{0} \\
& \text{if } b < 1, \mathbf{s} \leq \mathbf{0}
\end{aligned}
$$